Problem 31

Suppose that a mass m slides without friction on a horizontal surface. The mass is attached to a spring with spring constant k, as shown in Figure 3.7.10, and is also subject to viscous air resistance with coefficient γ. Show that the displacement $u(t)$ of the mass from its equilibrium position satisfies Eq. (21). How does the derivation of the equation of motion in this case differ from the derivation given in the text?

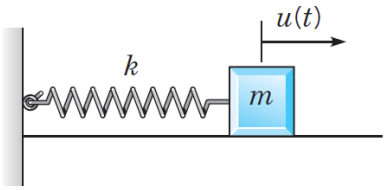
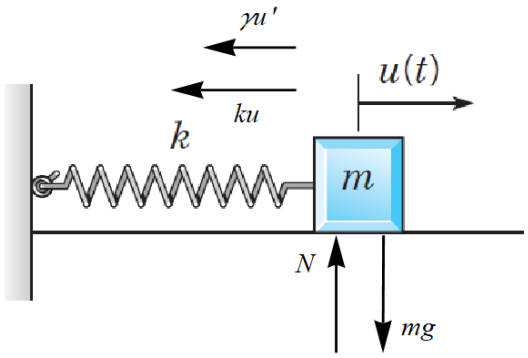


FIGURE 3.7.10 A spring–mass system.

Solution

Start by drawing a free-body diagram of the block. As a result of moving it to the right a distance u, two forces ku and $\gamma u'$ act to the left.



Apply Newton's second law in the u-direction.

$$\sum F_u = ma_u$$

$$-\gamma u' - ku = ma_u$$

Use the fact that the acceleration is the second derivative of position.

$$-\gamma u' - ku = mu''$$

Bring all terms to one side to obtain Eq. (21) in the text.

$$mu'' + \gamma u' + ku = 0 \tag{21}$$

By deriving the equation of motion for a block moving horizontally, it wasn't necessary to find the new equilibrium position due to gravity.